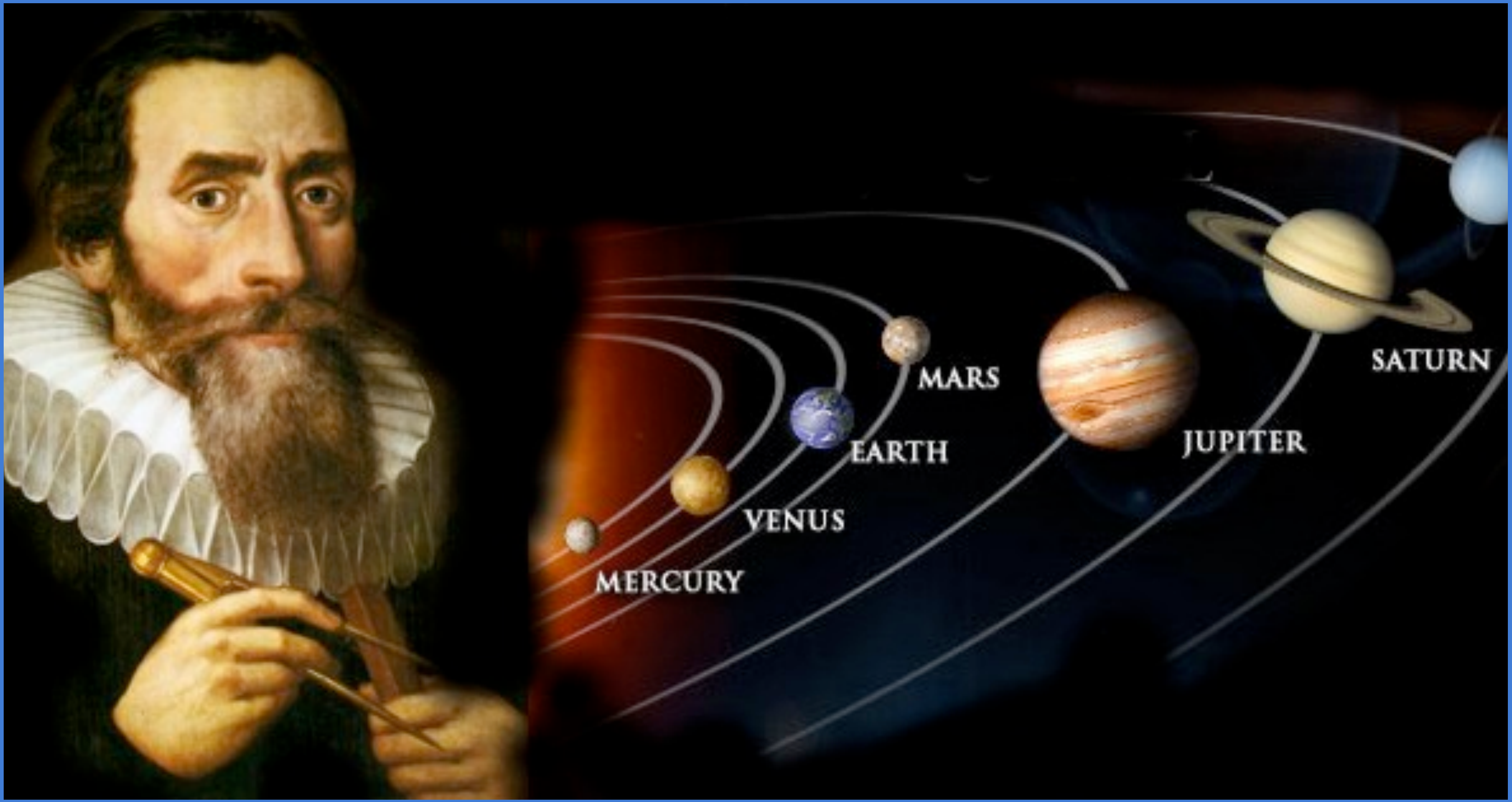
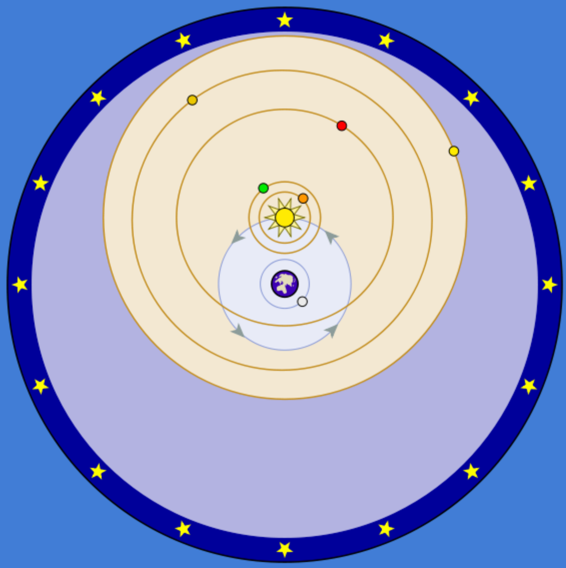
Kepler's Laws
&
Satellite Motion

Johannes Kepler (1571-1630)

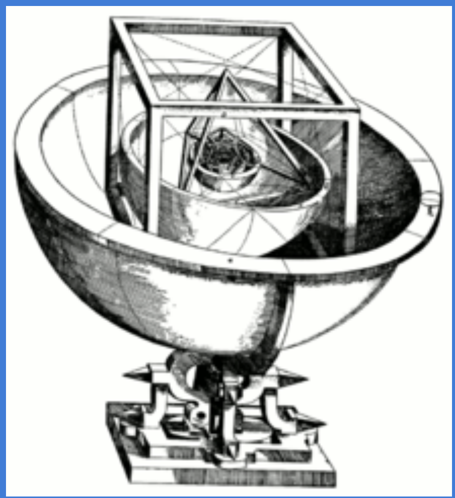


Tycho Brahe (1546 – 1601) built the first modern astronomical observatories. His instruments like the mural quandrant enabled him to measure the positions of stars and planets to an accuracy of about 1 minute. Before Tycho Brahe died, he hired Kepler as his assistant. Kepler tried unsuccsessfully to convert Brahe from the geocentric to the heliocentric model of the solar system.

Brahe's geocentric model



In 1595, Kepler published *Mysterium Cosmographicum* (*The Cosmographic Mystery*). He found that each of the five Platonic solids could be uniquely inscribed and circumscribed by spherical orbs; nesting these solids, each encased in a sphere, within one another would produce six layers, corresponding to the six known planets—Mercury, Venus, Earth, Mars, Jupiter, and Saturn.



By ordering the solids correctly—octahedron, icosahedron, dodecahedron, tetrahedron, cube—Kepler found that the spheres could be placed at intervals corresponding (within the accuracy limits of available astronomical observations) to the relative sizes of each planet's path, assuming the planets circle the Sun. Kepler also found a formula relating the size of each planet's orb to the length of its orbital period: from inner to outer planets, the ratio of increase in orbital period is twice the difference in orb radius. However, Kepler later rejected this formula, because it was not precise enough. Nevertheless, it laid the foundation for his later work now known as Kepler's 3rd Law.

The term *revolution* means to rotate around something, but it also means an upheaval or great change in ideas. This meaning of the word can be traced back 400 years ago to Nikolai Copernicus.

American Revolution, 1776

French Revolution, 1789

Thinking of the earth revolving around the sun instead rather than vice versa was a great change in how people thought about the solar system—indeed the universe—was constructed.

The sun-centered or heliocentric model of the solar system also went against the dogma of the Roman Catholic Church at the time—which not only exerted great influence on how governments operated but also on people's belief systems.

Rose Revolution (USSR), 1989

Tunisian Revolution, 2011

Indeed, Giordano Bruno—an Italian mathematician and astronomer— was burned at the stake in 1600 for advocating the Copernican geocentric model. Torture, brutality and intimidation are tactics still in use today by governments to coerce their populace—sometimes just to stay in power.

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$$\frac{r^3}{T^2} = k$$

Sir Issac Newton (1642 – 1727)

Although Kepler discovered what is now known theThree *Laws of Planetary Motion*, he could not explain *why* they were true. That did not come until years later from Issac Newton formulated the laws of motion that are the basis of mechanics—that are still valid today!

Sir Issac Newton

Newton formulated what is now known as his 2nd Law of Motion:

$$F_{net} = ma$$

Sir Issac Newton

This enabled him to formulate how objects are influenced (or attracted) in a gravitational field:

$$W = mg$$

Sir Issac Newton

He was also the first to identify the acceleration on objects forced to move in circles as:

$$a_c = \frac{v^2}{r}$$

Sir Issac Newton

And therefore the net force:

$$F_c = ma_c$$

$$= \frac{mv^2}{r}$$

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And finally what is perhaps the greatest intellectual discovery of all time—the *Law of Universal Gravitation:*

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This simple algebraic expression Mm/r^2 says how *everything in the universe* is related to *everything else*—a far-reaching statement indeed!

Although the orbits of the planets are ellipses, they are *very* close to circles. The gravitational pull of the sun provides the force that causes the planet to go in its nearly circular orbit.

$$F_c = F_g$$

The gravitational pull of the Sun provides the centripetal force of the satellite.

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Gravity provides the centripetal force of the satellite.

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

Recall from the *Flying Pig* lab that the tangential velocity of the pig is simply the circumference divided by period. The same is true for satellites in circular orbits:

$$v = \frac{2\pi r}{T}$$

Since $$v = \frac{2\pi r}{T}$$

We can square both sides:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2$$

$$= \frac{4\pi^2 r^2}{T^2}$$

Equating equivalent expressions for v^2:

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

The ratio of two measurable quantities—*radius* and *period*—equals a constant.

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$\frac{r^3}{T^2} = k$$

$$\frac{r^3}{T^2} = k_{Sun}$$

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If the distance of the planets to the sun are expressed in convenient units like astronomical units (1AU = the distance from the earth to the sun) and the period *T* is expressed in earth years, then the constant *k* equals 1!

But the same analysis for the planets orbiting the sun applies to moons orbiting Jupiter and can be extended to pairs of stars orbiting their common center of mass. This is how astronomers determine the mass of distant planets and stars.

$$M_{Sun} = \frac{4\pi^2}{G}\left(\frac{r^3_{Sun}}{T^2_{Sun}}\right)$$

$$M_J = \frac{4\pi^2}{G}\left(\frac{r^3_J}{T^2_J}\right)$$

$$m_1 + m_2 = \frac{4\pi^2}{G}\left(\frac{r^3_{Stars}}{T^2_{Stars}}\right)$$

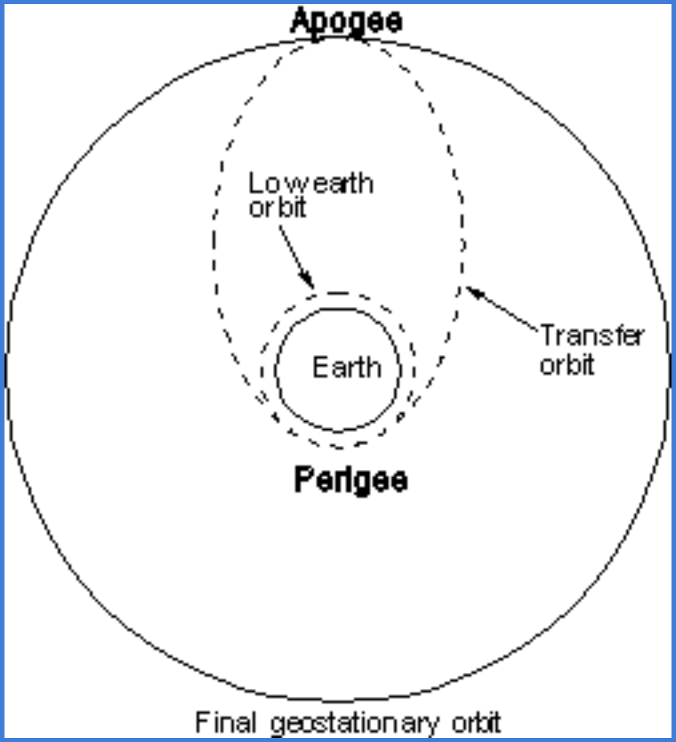
Geosynchronous Orbits

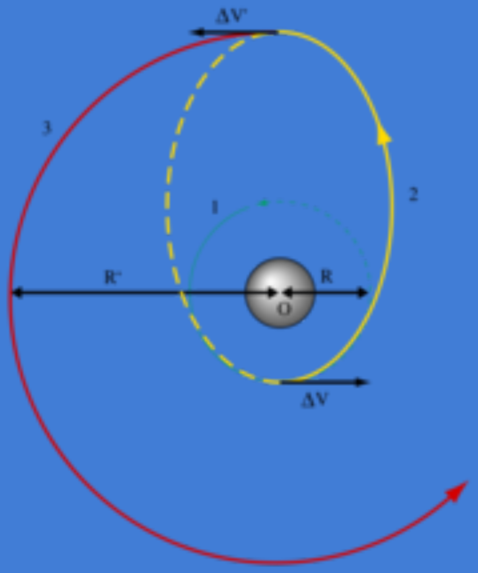
In 1945, British journalist Arthur C. Clarke who later become one of the most famous science fiction novelists of all time proposed that the new invention TV might be someday broadcast from satellites in so-called *geosynchronous orbits* (literally meaning earth-synchronized) from outer space—22,300 miles from the earth's surface. At this distance the orbital period of a satellite equals the rotational period—24 hours for us here on earth. Satellites in this position always appear above the earth in the same point in the sky. Dubbed unfeasible by some and impossible by others, he was largely ignored because of the great distances involved.

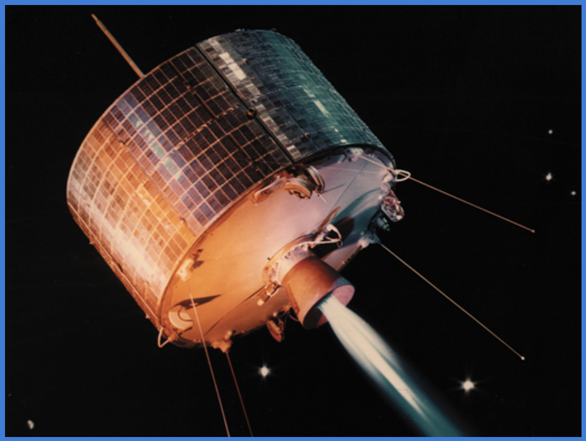
Geosynchronous Orbits

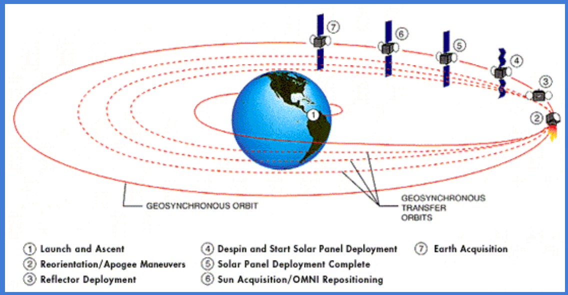
Rockets were in their infancy and commonly blew-up, so it was hard for anyone—including scientists to imagine satellites the size of cars at such distances. But by the early 1970’s rockets became more reliable. Now there are hundreds or even thousands of satellites in orbit—many of which such as weather, satellite TV (DirecTV and Dish TV), communications are in geosynchronous orbits.

Inserting Satellites in Geosynchronous Orbits









How to find *g* at a distance greater than the earth's surface:

$$F_{net} = F_g$$

$$mg = \frac{GMm}{r^2}$$

$$g = \frac{GM}{r^2}$$

Finding the Tangential Speed of the Satellite:

$$F_g = F_c$$

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{rg}$$

At the surface of the earth *r* is about 6400 km or 6,400,000 meters.

$$v = \sqrt{(6{,}400{,}000)(9.8)}$$

$$= 8000 \text{ m/s}$$

$$= 8 \text{ km/s}$$

Equating Equivalent Expressions for the Tangential Velocity and Solving for *r*:

$$v = \sqrt{rg} = \frac{2\pi r}{T}$$

$$v^2 = rg = \frac{4\pi^2 r^2}{T^2}$$

$$r = g\left(\frac{T^2}{4\pi^2}\right)$$

Substitute the value of *g* at the geosynchronous orbit and solve for *r*:

$$r = \left(\frac{GM}{r^2}\right)\left(\frac{T^2}{4\pi^2}\right)$$

$$r^3 = \frac{GM}{4\pi^2}\left(T^2\right)$$

$$r = \sqrt[3]{\frac{GM}{4\pi^2}\left(T^2\right)}$$

Geosynchronous Orbits

Substituting the known values for the universal gravitational constant, *G*, the mass of the earth, *M*, and the number of seconds in a year, *T*, the distance is:

$$r = \sqrt[3]{\frac{GM}{4\pi^2}\left(T^2\right)}$$

$$r = \text{ 22,300 miles above the earth's surface}$$

Appendix A: Using the Computer to Graph Planetary Data

- The computer is a powerful analytical tool. It can be used to show how two quantities are related. Typically, takes 10 minutes for students to manipulate the powers to make the graph a straight line using a spreadsheet—Kepler spent 10 *years!* The significance of a linear graph is whatever you're graphing along the vertical axis is proportional to whatever you're graphing along the horizontal axis. Since the graph of r^3 vs. T^2 is a straight line, $r^3 \sim T^2$, or

$$\frac{r^3}{T^2} = k$$

The slope of the graph of log of R vs. T is the functional relationship between the variables

$$\frac{r^3}{T^2} = k$$

$$r^3 = kT^2$$

$$\log\left(r^3\right) = \log\left(kT^2\right)$$

$$3\log r = 2\log T + \log k$$

$$\log r = \frac{2}{3}\log T + \frac{1}{3}\log k$$

$$y = mx + b$$

$$\therefore m = \frac{2}{3}$$

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Therefore the slope tells us how *R* and *T* are related.